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Three-dimensional quantum electrodynamics as an effective interaction*

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Abstract

We obtain a Quantum Electrodynamics in 2+1 dimensions by applying a Kaluza–Klein type method of dimensional reduction to Quantum Electrodynamics in 3+1 dimensions rendering the model more realistic to application in solid-state systems, invariant under translations in one direction. We show that the model obtained leads to an effective action exhibiting an interesting phase structure and that the generated Chern–Simons term survives only in the broken phase.

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1. Introduction

Models in (2+1) dimensions with or without Chern–Simons terms have been a subject of an intense theoretical study in the last years[1−9], for several important reasons. First, they are generalizations of two-dimensional models[10], where one can have an interesting theoretical laboratory for check of ideas, a little more complex than the two-dimensional case, but still without all the technical complexity of four dimensions. This is the case of the gravitation theory in (2+1) dimensions[1] that is very simple from the classical point of view: it has zero degrees of freedom, but topological effects appear, and are effective to the understanding of the physics involved. Another equally important to study such models, is that they can be useful in determining properties of certain important practical condensed-matter systems. In particular many efforts have been recently devoted to find good theoretical explanations for the high-T_c superconductivity[6−9,11,12] as well as for the fractional quantum Hall effect[13] which are observed in some layered ceramics. These materials exhibit a planar geometry that can, in principle, be described by models in quantum field theory with only two spatial coordinates. So one of the main challenges in this area is to find correct models for description of these phenomena.

In the present work we obtain a special model, the quantum electrodynamics in (2+1) dimensions with a Chern–Simons term, starting from the quantum electrodynamics in (3+1)-dimensional space time (QED_4), which we believed to be a good choice as starting point since it is always the most realistic theory when the effects previously referred to are treated as in the usual way. We obtain the model via a Kaluza–Klein-type mechanism for periodic fields[14−17], in which a dimensional reduction of extra coordinate is made. Such an extra dimension is associated to the direction of the layers in the sample in the same way as Bloch wave functions are found in solid-state systems, and we are naturally led to a compact space (one torus in this case) such that the three-dimensional physics is a low energy approximation of a broader theory. Indeed such reductions are commonly used in condesend-matter systems such as the fractional Hall effect, where the edges currents will be described by an even simpler model, in two-dimensional space-time.(See,for instance, Ref.[18] and the list of references therein.) We also analyse in this work the phase structure of the three-dimensional quantum electrodynamics obtained(KKB-QED_3) with special attention being paid to the role played by the Chern–Simons term.

The paper is organized as follows: In section 2 we start with the four-dimensional quantum electrodynamics and obtain a theory in three dimensions via the above mentioned Kaluza–Klein scheme for periodic fields(KKB). In section 3 we obtain the effective potential for this three-dimensional quantum electrodynamics and find its minimum. In section 4 we investigate the phase structure of the model and calculate the Chern–Simons term induced by the fermions. Section 5 contains our conclusions.

2. The KKB technique and the model

In order to obtain a model in three dimensions, we start with the quantum electrodynamics in (3+1) dimensions described by the Lagrangian

$$\mathcal{L} = \overline{\Psi} i D\!\!\!/ \Psi - M\overline{\Psi}\Psi - \frac{1}{4}F_{mn}F^{mn} \quad , \tag{2.1}$$

where

$$iD\!\!\!/ = \Gamma^m(i\partial_m + eA_m) = i\Gamma^m D_m \quad , \quad m,n = 0,1,2,3 \quad ,$$
$$iD_\mu = i\partial_\mu + eA_\mu \quad , \quad \mu = 0,2,3 \quad .$$

M and e are the mass and the electric charge in four dimensions and Γ^m are the Dirac gamma matrices. The gauge field strength F^{mn} is given in terms of the vector potential A_m by $F^{mn} = \partial^m A^n - \partial^n A^m$. We employ the chiral representation of Dirac Γ matrices, i.e.,

$$\Gamma^0 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \quad , \quad \vec{\Gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix} \quad .$$

They satisfy the Clifford algebra $\{\Gamma^m, \Gamma^n\} = 2G^{mn}$, with the metric tensor $G^{mn} = (+---)$.

The Dirac spinor is given in terms of the Pauli spinors, by

$$\Psi = \begin{pmatrix} \psi \\ \chi \end{pmatrix} \quad , \quad \overline{\Psi} = \psi^\dagger \Gamma^0 = (\, -\chi^\dagger \quad -\psi^\dagger \,) \quad .$$

In terms of the three-dimensional quantities, now represented by greek indices ($\mu, \nu = 0,2,3$) and small letters, the Lagrangian is rewritten, as

$$\begin{aligned} \mathcal{L} = & i\overline{\psi}\gamma^\mu D_\mu \psi + i\overline{\chi}\gamma^\mu D_\mu \chi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial^\mu A_1 \partial_\mu A_1 + \\ & + \frac{1}{2}(\partial_1 A_0)^2 - \frac{1}{2}(\partial_1 A_2)^2 - \frac{1}{2}(\partial_1 A_3)^2 - \partial_1 A_0 \partial_0 A_1 + \partial_1 A_3 \partial_3 A_1 + \\ & + \partial_1 A_2 \partial_2 A_1 + M(\overline{\chi}\psi + \overline{\psi}\chi) + e(\overline{\psi}\psi A_1 - \overline{\chi}\chi A_1) + (\overline{\psi}\partial_1\psi - \overline{\chi}\partial_1\chi) \quad , \end{aligned} \tag{2.2}$$

where the index 1 is compactified.

Above, the representation of Dirac γ matrices in three dimensions has Pauli matrices given by

$$\gamma_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \gamma^{"5"} = \sigma^1 \quad ; \quad \gamma_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = -i\sigma^3 \quad ; \quad \gamma_3 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} = i\sigma^2 \quad ,$$

which, in particular, satisfy the useful property $\gamma^\mu\gamma^\nu = g^{\mu\nu} - i\epsilon^{\mu\nu\rho}\gamma_\rho$. Where $\epsilon_{\mu\nu\rho}$ is the Levi-Civita tensor which, in three dimensions, is totally antisymmetric.

Now we implement the KKB scheme[14] by considering that the model admits a spontaneous compactification[14–17] on a Minkowski (2+1)-dimensional space-time and G/H a

homogeneous compact space associated to the extra coordinate (one torus in this case) that is assumed to be a circle of length $l_x = l$. Physically, this is equivalent to suppose that we are dealing with a sample with layers, whose interspace l is very small, and physics is periodic in $x \rightarrow x + l$. In condensed-matter systems, l is the cristal interlayer space. We split the four components of the four-vector position in $x_m = (x_\mu, x_1) = (x_\mu, x)$ and make the decomposition of the vector gauge field in $A_m \rightarrow (A_\mu, A)$, where we denote $A_x = A_1 = A$.

Furthermore, we also assume:

$$
\begin{aligned}
\psi(x_\mu, x + l) &= \psi(x_\mu, l) \quad , \\
\chi(x_\mu, x + l) &= \chi(x_\mu, x) \quad , \\
A(x_\mu, x + l) &= A(x_\mu, x) \quad .
\end{aligned}
$$

The above relations break the (3+1)-dimensional Poincaré invariance of (2.1), P_{3+1} to a (2+1)-dimensional Poincaré invariance times $U(1)$, that is, $P_{3+1} \rightarrow P_{2+1} \times U(1)$.

Now ψ, χ and A can be expanded in Fourier series (we are admitting planar structures in specified matter-condensed systems of interest in which the Bloch theorem is valid).

$$
\begin{aligned}
\psi(x_\mu, x) =& \frac{1}{\sqrt{l}} \sum_r \psi^{(r)} \mathrm{e}^{\frac{irx}{l}} \quad , \\
\chi(x_\mu, x) =& \frac{1}{\sqrt{l}} \sum_r \chi^{(r)} \mathrm{e}^{\frac{irx}{l}} \quad , \\
A_\mu(x_\mu, x) =& \frac{1}{\sqrt{l}} \sum_r A_\mu^{(r)} \mathrm{e}^{\frac{irx}{l}} \quad ,
\end{aligned}
\tag{2.3}
$$

with r integer.

By substituting the above expansions in the Lagrangian (2.2) we can integrate over x from zero to l and obtain a three-dimensional description of this theory through the reduced Lagrangian.

$$
\begin{aligned}
\widetilde{\mathcal{L}} = & i\overline{\psi}\gamma^\mu D_\mu \psi + i\overline{\chi}\gamma^\mu \tilde{D}_\mu \chi + M(\overline{\chi}\psi + \overline{\psi}\chi) + qA(\overline{\psi}\psi - \overline{\chi}\chi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \partial^\mu A \partial_\mu A - \\
& - \sum_r m_{(r)} \left(\overline{\psi}^{(-r)} \psi^{(r)} - \overline{\chi}^{(-r)} \chi^{(r)} \right) - i \sum_r m_{(r)} \left(A^{(-r)} \partial^\mu A_\mu^{(r)} \right) + \frac{1}{2} \sum_r m_{(r)}^2 A_\mu^{(-r)} A^{\mu(r)}
\end{aligned}
\tag{2.4}
$$

where $m_{(r)}^2 = \frac{r^2}{l^2}$, and the momentum of χ is opposite to that of ψ (see notation after eq. (3.5)).

The Lagrangian $\widetilde{\mathcal{L}}$ is adentical to $\mathcal{L}$ up to some rescaling of the coupling constants and the fields in order to give them the canonical dimensions of the QED_4. These rescaling factors disappear after integration over x. Notice that there are two limiting cases: if $l \rightarrow \infty$, the original QED_4 is recovered; if $l \rightarrow 0$ ($l \sim$ size of the compact space or torus),

we are making, in fact, an ordinary dimensional reduction in which the fields do not depend on the extra-coordinate x. In condensed-matter systems with layered structures, such as high-T_c superconductors, the truncation by letting $l \to 0$ is very reasonable since l will have to be related to the interlayers distance ($\sim 10^{-9}$m)and to the effective mass $m \sim \frac{1}{l}$. In these cases, $l << l_y$ and $l << l_z$.

3. The effective potential in the KKB-QED$_3$

In order to study the physics of the KKB-QED$_3$ with fermions inducing a parity-violating Chern–Simons term we now calculate the effective action upon integration over the fermions. The term

$$\sum_r m_{(r)} \left(\overline{\psi}^{(-r)}\psi^{(r)} - \overline{\chi}^{(-r)}\chi^{(r)}\right) \quad ,$$

can be neglegted since in three dimensions an even number of fermions can be paired to form Dirac fermions with parity-conserving mass terms[4]. The two last terms can be dropped because when we make calculations in high orders in perturbation theory all higher-order graphs in the effective action are proportional to powers of $\frac{1}{m}$ or $\frac{1}{r/l}$ and they vanish when $m \to \infty$, or $l \to 0$.

Thus, our reduced Lagrangian becomes, in this approximation

$$\begin{aligned}\mathcal{L} =& i\overline{\psi}\gamma^\mu\partial_\mu\psi + i\overline{\chi}\gamma^\mu\tilde{\partial}_\mu\chi + M(\overline{\chi}\psi + \overline{\psi}\chi) + qA(\overline{\psi}\psi - \overline{\chi}\chi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &+ \frac{1}{2}\partial^\mu A\partial_\mu A + q\left(\overline{\psi}\gamma_\mu\psi A^\mu + \overline{\chi}\gamma_\mu\chi A^\mu\right) \quad ,\end{aligned} \tag{3.1}$$

where q is an effective electric charge in three dimensions. Now, we may work with the above Lagrangian to obtain, in perturbation theory, a renormalized effective gauge field action which contains a Chern–Simons induced by fermions. This term will have to appear in the effective action when ultraviolet divergences are regulated in a gauge-invariant way, for example, using the Pauli-Villars regularization. Only the vacuum polarization and triangle graphs are ultraviolet divergent. But in the Abelian case, only the first one requires regularization.

Thus, we integrate over the fermions fields and calculate the effective potential $V_{eff}(A)$ at ono-loop order. Before, making the substitution $A \to A' = A + a$, where $a = \langle 0|A|0\rangle = \langle A\rangle$ we have

$$\begin{aligned}\mathcal{L}_{eff} =& i\overline{\psi}\gamma^\mu\partial_\mu\psi + i\overline{\chi}\gamma^\mu\tilde{\partial}_\mu\chi + M(\overline{\chi}\psi + \overline{\psi}\chi) + qA(\overline{\psi}\psi - \overline{\chi}\chi) + qa(\overline{\psi}\psi - \overline{\chi}\chi) \\ &- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial^\mu A\partial_\mu A + q\left(\overline{\psi}\gamma_\mu\psi + \overline{\chi}\gamma_\mu\chi\right)A^\mu \quad ,\end{aligned} \tag{3.2}$$

The effective potential $V_{eff}(a) = i\ln\det\mathcal{O}$, or, in a more conveniente way,

$$V_{eff} = -ie^{\mathrm{tr}\ln\mathcal{O}} \quad , \tag{3.3}$$

where

$$\mathcal{O} = \begin{pmatrix} i\partial - qa & M \\ M & i\partial + qa \end{pmatrix} = \begin{pmatrix} k_0 - \vec{k}\cdot\vec{\sigma} - \mu\sigma_1 & M\sigma_1 \\ M\sigma_1 & k_0 + \vec{\sigma}\cdot\vec{k} + \mu\sigma_1 \end{pmatrix} \quad . \tag{3.4}$$

After some algebra, V_{eff} is given by

$$\begin{aligned} V_{eff}(a) = & -\frac{i}{2}\mathrm{tr}\Big[\int \frac{\mathrm{d}^3k}{(2\pi)^3}\ln\left(1 + \frac{(qa)^2}{(k^2 - M^2)}\right)1- \\ & -\int \frac{\mathrm{d}^3k}{(2\pi)^3}\sum_n \frac{(qa)^{2n+1}}{(2n+1)}\frac{1}{(k^2 - m^2)}\begin{pmatrix} \not{k} & M \\ M & \tilde{\not{k}} \end{pmatrix}\Big] \quad , \end{aligned} \tag{3.5}$$

where $\not{k} = k_0 - \vec{\sigma}\cdot\vec{k}$ and $\tilde{\not{k}} = k_0 + \vec{\sigma}\cdot\vec{k}$.

Making a Wick rotation, and taking the trace, we have

$$V_{eff} = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\ln\left(1 - \frac{(qa)^2}{(k^2 + M^2)}\right) \quad , \tag{3.6}$$

and it can be rewritten in terms of a cutoff Λ

$$\begin{aligned} V_{eff}(a) = & \frac{1}{2\pi^2}\int_0^\Lambda \mathrm{d}k k^2\left[\ln\left(k^2 + M^2 - (qa)^2\right) - \ln\left(k^2 + M^2\right)\right] \\ = & -\frac{1}{2\pi^2}\left[\Lambda(qa)^2 - \frac{\pi}{3}(qa)^3\right] + \mathcal{O}\left(\frac{1}{\Lambda}\right) \quad . \end{aligned} \tag{3.7}$$

Notice that, above, the cutoff Λ can be removed through a renormalization.

By analysing the above effective potential, we obtain two values that make it minimum $\langle qa\rangle = qa = \mu = 0$ and $\mu = \frac{2\Lambda}{\pi}$. Therefore, our system, described by a reduced quantum electrodynamics, the KKB-QED$_3$, exhibits a phase transition, and there is a phase in which the field acquires mass. Furthermore, if we integrate out completely the A field, we find also a Gross-Neveu interaction in three dimensions.

4. The Chern–Simons term and the phase structure

Now that we have shown the existence of two phases in our theory, the next question to address is whether a Chern–Simons term is a given phase and which is its role.

First we find the fermion propagators of the KKB-QED$_3$. Formally, in terms of the path integral with respect to the fields $\psi, \overline{\psi}, \chi, \overline{\chi}, A_\mu$ and A, the generating functional is

$$\mathcal{Z}[J, \overline{J}, \eta, \overline{\eta}, K, j] = \int \mathcal{D}\overline{\psi}\mathcal{D}\psi\mathcal{D}\overline{\chi}\mathcal{D}\chi\mathcal{D}A_\mu\mathcal{D}A\, \mathrm{e}^{i\int \mathrm{d}^3x\left[\mathcal{L}_{eff} + (\overline{J}\psi + \overline{\psi}J + \overline{\chi}\eta + \overline{\eta}\chi)\right]} \quad , \tag{4.1}$$

where $\mathcal{L}_{eff}$ is that one given by eq. (3.2) and $\overline{J}, J, \eta, \overline{\eta}, K$ and j are external c-numbers sources coupled to the fields. The propagators read

$$
\begin{aligned}
\langle\psi\overline{\psi}\rangle &= -\left.\frac{\delta^2\mathcal{Z}}{\delta J\delta\overline{J}}\right|_{J=\overline{J}=0} = S_{11}(k) = i\frac{\not{k}-\mu}{k^2-(M^2+\mu^2)} \quad ,\\
\langle\chi\overline{\chi}\rangle &= -\left.\frac{\delta^2\mathcal{Z}}{\delta\eta\delta\overline{\eta}}\right|_{\eta=\overline{\eta}=0} = S_{22}(k) = i\frac{\tilde{\not{k}}+\mu}{k^2-(M^2+\mu^2)} \quad ,\\
\langle\psi\overline{\chi}\rangle &= -\left.\frac{\delta^2\mathcal{Z}}{\delta\overline{J}\delta\eta}\right|_{\overline{J}=\eta=0} = S_{12}(k) = i\frac{M}{k^2-(M^2+\mu^2)} \quad ,\\
\langle\chi\overline{\psi}\rangle &= -\left.\frac{\delta^2\mathcal{Z}}{\delta\overline{\eta}\delta J}\right|_{\overline{\eta}=J=0} = S_{21}(k) = i\frac{M}{k^2-(M^2+\mu^2)} \quad .
\end{aligned}
\tag{4.2}
$$

Defining $m_t^2 = \mu^2 + M^2$, we compute below all contributions to the effective action from vacuum polarization graphs, taking into account the above propagators.

We can write the vacuum polarization as

$$
\Pi^{\mu\nu} = \Pi^{\mu\nu}_{11} + \Pi^{\mu\nu}_{22} + \Pi^{\mu\nu}_{12} + \Pi^{\mu\nu}_{21} \quad . \tag{4.3}
$$

The graphs involving only one kind of fermion (ψ or χ) give, for example

$$
\Pi^{\mu\nu}_{11} = iq^2 \int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{\mathrm{tr}\,[\gamma^\mu(\not{k}-\mu)\gamma^\nu(\not{k}+\not{p}-\mu)]}{(k^2-m_t^2)[(k+p)^2-m_t^2]} \quad . \tag{4.4}
$$

Upon evaluating the trace over Dirac matrices and using the same regularization prescription of ref. [3] and [4], we obtain a contribution for the Chern–Simons (CS) term only from

$$
\Pi^{\mu\nu}_{11\,CS} = \Pi^{\mu\nu}_{22\,CS} = \frac{-\mu}{\sqrt{M^2+\mu^2}}\frac{q^2}{4\pi}(i\epsilon^{\mu\nu\rho}p_\rho) \quad . \tag{4.5}
$$

This way, we have a Chern–Simons term generated that can be expressed in the effective Lagrangian of the KK-QED$_3$ model. The effective Lagrangian is given by the expression

$$
\begin{aligned}
\widetilde{\mathcal{L}}_{eff} =& i\overline{\psi}\gamma^\mu\partial_\mu\psi + i\overline{\chi}\gamma^\mu\tilde{\partial}_\mu\chi + M(\overline{\chi}\psi+\overline{\psi}\chi) + \mu(\overline{\psi}\psi-\overline{\chi}\chi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\\
&+ \frac{1}{2}\partial^\mu A\partial_\mu A + q\left(\overline{\psi}\gamma_\mu A^\mu\psi + \overline{\chi}\gamma_\mu A^\mu\chi\right) + \frac{q^2}{4\pi}\frac{\mu}{|m_t|}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho + \mathcal{O}\left(\frac{1}{m_t}\right) \quad ,
\end{aligned}
\tag{4.6}
$$

By analysing this special Chern–Simons term, we notice that it vanishes when the system is in the phase where the A field is massless (either with massive or massless fermions). However, at the broken phase, this term becomes very important in the dynamics of the KK-QED$_3$ because it does not vanish even when the fermions are massive, except of course, if $M \to \infty$. When $M = 0$, we have $m_t = \mu$ and there will be a $\pm$ sign from $\mu/|m_t|$.

We can use eq. (4.6) to derive the vector current in terms of the Chern–Simons term. We have

$$q\langle j^\mu(x)\rangle = q\langle\overline{\psi}_i\gamma^\mu\psi_i\rangle = -\frac{\delta\widetilde{\mathcal{L}}_{eff}}{\delta A_\mu} \quad , \tag{4.7}$$

with $i = 1, 2$, where $\psi_1 = \psi$ and $\psi_2 = \chi$. Now

$$qj^\mu = q\langle\overline{\psi}_i\gamma^\mu\psi_i\rangle = \partial_\nu F^{\mu\nu} + \mathbb{C}\epsilon^{\mu\nu\rho}F_{\nu\rho} \quad , \tag{4.8}$$

where $\mathbb{C} = \frac{q^2\mu}{4\pi\sqrt{M^2+\mu^2}}$ is the Chern–Simons coefficient. We rewrite the last equation as

$$qj^\mu = (\Box g^{\mu\nu} + 2\mathbb{C}\partial_\rho\epsilon^{\mu\nu\rho})\, A_\nu \quad . \tag{4.9}$$

In momentum space, we have

$$qj^\mu = -\left(p^2 g^{\mu\nu} + 2i\mathbb{C}p_\rho\epsilon^{\mu\nu\rho}\right) A_\nu \quad . \tag{4.10}$$

Now, we write the A_μ field in terms of the current $\overline{\psi}_i\gamma^\mu\psi_i$ inverting the above expression, then

$$A_\mu = q\left[\left(\frac{1}{4\mathbb{C}^2 - p^2}\right) g_{\mu\rho} + \frac{2i\mathbb{C}p^\nu}{p^2(4\mathbb{C}^2 - p^2)}\epsilon_{\mu\nu\rho} + p_\mu p_\rho\right] \left(\overline{\psi}_i\gamma^\rho\psi_i\right) \quad . \tag{4.11}$$

Substituting (4.11) in (4.6), we obtain

$$\begin{aligned}\widetilde{\mathcal{L}}_{eff} =\ & i\overline{\psi}\gamma^\mu\partial_\mu\psi + i\overline{\chi}\gamma_\mu\tilde{\partial}^\mu\chi + M(\overline{\chi}\psi + \overline{\psi}\chi) + \mu(\overline{\psi}\psi - \overline{\chi}\chi) \\ & + q^2(\overline{\psi}\psi - \overline{\chi}\chi)\Box^{-1}(\overline{\psi}\psi - \overline{\chi}\chi) + q^2(\overline{\psi}_i\gamma^\mu\psi_i)M_{\mu\nu}(\overline{\psi}_j\gamma^\nu\psi_j) \quad ,\end{aligned} \tag{4.12}$$

where

$$M_{\mu\nu} = \frac{1}{4\mathbb{C}^2 - p^2}\left(g_{\mu\nu} - 2i\mathbb{C}\frac{p^\rho}{p^2}\epsilon_{\mu\nu\rho}\right) \quad . \tag{4.13}$$

If we are now interested in the low-momentum behaviour of the theory, we must consider at small momentum ($p \to 0$) each term of eq. (4.12). Thus, we can write the low-momentum effective Lagrangian as

$$\begin{aligned}\widetilde{\mathcal{L}}_{eff} = & \, i(\overline{\psi}\,\partial\!\!\!/\psi + \overline{\chi}\tilde{\partial\!\!\!/}\chi) + M(\overline{\chi}\psi + \overline{\psi}\chi) + \mu(\overline{\psi}\psi - \overline{\chi}\chi) + \\ & + q^2(\overline{\psi}\psi - \overline{\chi}\chi)\Box^{-1}(\overline{\psi}\psi - \overline{\chi}\chi) + \frac{q^2}{4\mathbb{C}^2 - p^2}(\overline{\psi}\gamma^\mu\psi + \overline{\chi}\gamma^\mu\chi)(\overline{\psi}\gamma_\mu\psi + \overline{\chi}\gamma_\mu\chi) .\end{aligned} \tag{4.14}$$

Notice that the above effective Lagrangian displays an interesting structure. It contains two types of fermions besides non-local and quartic terms.

We believe that this new effective Lagrangian opens up a promising possibility of getting a good description of anyonic superconductivity and superfluity. It is also important

to notice that the current (4.8) calculated in terms of the Chern–Simons term can be used to get the Hall conductivity of the vacuum, i.e.

$$j^0 \sim \mathbb{C}\epsilon^{0ij}F_{ij} = 2\mathbb{C}F_{23} = 2\mathbb{C}B \quad , \tag{4.15}$$

whereas

$$j^2 \sim 2\mathbb{C}\epsilon^{203}F_{03} = i\sigma_{23}E^2 \quad , \tag{4.16a}$$

$$j^3 = 2\mathbb{C}\epsilon^{302}F_{02} \quad . \tag{4.16b}$$

In the limit $\mu \to \infty$ we find $\sigma = \frac{q^2}{2\pi}$.

5. Conclusions

In this article we have investigated the implementation of the KKB scheme within the framework of the quantum electrodynamics in four dimensions. We performed a dimensional reduction, that is, a compactification of an extra spatial coordinate and obtained a reduced theory, the QED_3, which exhibits an interesting structure. We shown that the system described for this model undergoes a phase transition and, in addition, in the broken phase, in which A field acquires mass, a Chern–Simons term survives. Moreover We are also naturally led to fractional statistics since the Thirring interaction can be bosonized in three dimensions[19]. Actually, this fact opens up a possibility of arriving at similar conclusions to those drawn for $(1+1)$-dimensional models, such as the chiral Gross–Neveu model where a bound state structure exists, such that fermions are solitons formed as bound states of other fields of the theory, leading to possible solution to fractional quantum Hall effect. We have calculated the vector current for the model and have used it to get a new effective Lagrangian in terms of the fermions fields of the theory. The low-momentum behavior of the Lagrangian is such that it describes some special four-fermions interactions. This is a good indication of its applicability in the theoretical studies on the basic phenomena of anyonic superconductivity and Quantum Hall Effect as used in Ref. [18]. The vector current associated to the two types of fermions of the theory was also used to get a Hall conductivity of the vacuum. The non-relativistic quantum mechanics and the nature of the fermion-fermion interactions of the KKB-QED_3 model are being presently investigated . Some results in this direction have been found recently considering the issue of symmetry breaking in a Gross–Neveu model with Thirring interaction.[20]

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